Experiment

Study of an LCR resonant circuit

1 Objectives

To study the behavior of a series LCR resonant circuit and to estimate the resonant frequency and Q-factor.

2 Circuit components/equipment

1. Capacitor
2. Resistors
3. Inductor
4. Function generator
5. Oscilloscope
6. Multimeter/LCR met
7. Connecting wires
8. Breadboard

3 Overview

Circuits containing an inductor L, a capacitor C, and a resistor R, have special characteristics useful in many applications. Their frequency characteristics (impedance, voltage, or current vs. frequency) have a sharp maximum or minimum at certain frequencies. These circuits can hence be used for selecting or rejecting specific frequencies and are also called tuning circuits. These circuits are therefore very important in the operation of television receivers, radio receivers, and transmitters.

Let an **alternating voltage** V_i be applied to an inductor L, a resistor R and a capacitor C all in series as shown in the circuit diagram. If I is the instantaneous current flowing through the circuit, then the applied voltage is given by

$$V_i = V_{R_{\mathrm{DC}}} + V_L + V_C = I\left(R_{\mathrm{DC}} + j\omega L - \frac{j}{\omega C}\right) \tag{1}$$

Here R_{DC} is the total DC resistance of the circuit that includes the resistance of the pure resistor, inductor and the internal resistance of the source. This is the case when the resistance of the inductor and source are not negligible as compared to the load resistance R. So, the total impedance is given by

$$R_{\mathrm{DC}} + j\omega L - \frac{j}{\omega C} \tag{2}$$

The magnitude and phase of the impedance are given as follows:

$$|Z| = \left[R_{DC}^2 + \left(\omega L - \frac{1}{\omega C} \right)^2 \right]^{1/2} \tag{3}$$

$$\tan\phi = \frac{\left(\omega L - \frac{1}{\omega C} \right)}{R_{DC}} \tag{4}$$

Thus, three cases arise from the above equations:

1. $\omega L > (1/\omega C)$, then $\tan\phi$ is positive and applied voltage leads current by phase angle ϕ.
2. $\omega L < (1/\omega C)$, then $\tan\phi$ is negative and applied voltage lags current by phase angle ϕ.
3. $\omega L = (1/\omega C)$, then $\tan\phi$ is zero applied voltage and current are in phase.

Here $V_L = V_C$, the circuit offers minimum impedance which is purely resistive. Thus, the current flowing in the circuit is maximum (I_o) and also V_R is maximum, and $V_{LC} = (V_L + V_C)$ is minimum. This condition is known as resonance and the corresponding frequency as resonant frequency (ω_o) expressed as follows:

$$\omega_o = \frac{1}{\sqrt{LC}} \quad \text{or} \quad f_o = \frac{1}{2\pi\sqrt{LC}} \tag{5}$$

At resonant frequency, since the impedance is minimum, hence frequencies near f_o are passed more readily than the other frequencies by the circuit. Due to this reason LCR-series circuit is called acceptor circuit. The band of frequencies which is allowed to pass readily is called passband. The band is arbitrarily chosen to be the range of frequencies between which the current is equal to or greater than $I_o/\sqrt{2}$. Let f_1 and f_2 be these limiting values of frequency. Then the width of the band is BW= $f_2 - f_1$.

The **selectivity** of a tuned circuit is its ability to select a signal at the resonant frequency and reject other signals that are close to this frequency. A measure of the selectivity is the **quality factor** (Q), which is defined as follows:

$$Q = \frac{f_o}{f_2 - f_1} = \frac{\omega_o L}{R_{DC}} = \frac{1}{R_{DC}\omega_o C} \tag{6}$$

In this experiment, you will measure the magnitude and phase of V_R and V_{LC} with respect to V_i which is $|(V_R/V_i)|, |(V_{LC}/V_i)|, \phi_R$ and ϕ_{LC} in the vicinity of resonance using the following working formulae

$$\left|\frac{V_R}{V_i}\right| = \frac{R}{|Z|} \tag{7}$$

$$\text{or, } \phi_R = -\arctan\left(\frac{\omega L - \frac{1}{\omega C}}{R_{DC}}\right) \tag{8}$$

$$\text{or, } \left|\frac{V_R}{V_i}\right| = \frac{\omega L - \frac{1}{\omega C}}{|Z|} \tag{9}$$

$$\text{or, } \phi_{LC} = -\arctan\left(\frac{R_{DC}}{\omega L - \frac{1}{\omega C}}\right) \tag{10}$$

4 Circuit diagram

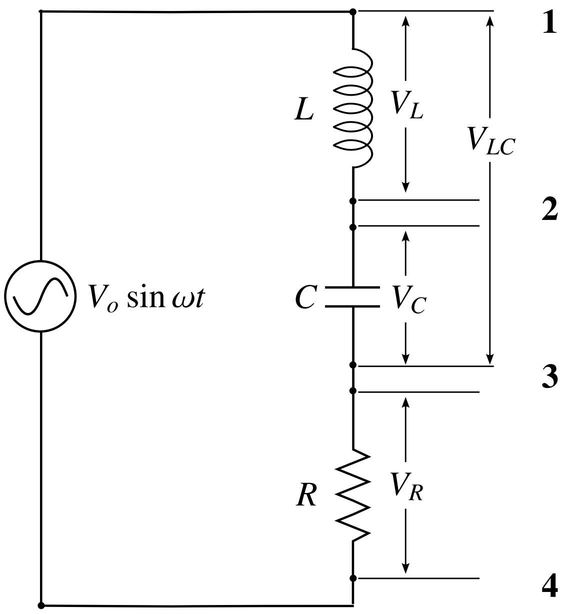

Figure 1: Circuit diagram of an LCR series resonant circuit.

5 Procedure

Measuring V_R, V_{LC}, ϕ_R and ϕ_{LC}

1. Using the multimeter/LCR meter, note down all the measured values of the inductance, capacitance and resistance of the components provided. Also, measure the resistance of the inductor. Calculate the resistance of the circuit. Calculate the resonant frequency.

2. Configure the circuit on a breadboard as shown in circuit diagram. Set the function generator frequency range in 10 - 20 kHz and function in sinusoidal mode. Set an input peak-to-peak voltage of 5V (say) with the oscilloscope probes.

3. Feed terminals 1 & 4 shown in the circuit diagram to oscilloscope channel 1 to measure input voltage V_i and terminal 3 & 4 to oscilloscope channel 2 to measure output voltage V_R, respectively. Note that terminal 4 is connected to the ground pin of the function generator and oscilloscope. Keep the settings such that you can measure f, V_i, V_R and ϕ simultaneously.

4. Vary the frequency in the set region slowly and record V_R and V_i (which may not remain constant at the set value). Read the frequency from oscilloscope. Also, measure the phase shift angle ϕ_R with proper sign.

5. Replace the resistor with another value and repeat steps 3. and 4. No phase measurement is required for the second resistor.

6. Now, interchange the probes of the function generator and oscilloscope, i.e. make terminal 1 as the common ground so that you will measure V_{LC} output across terminal 3 & terminal 1 and input voltage (V_i) across terminal 4 and terminal 1. Repeat step (d) to record V_{LC}, V_i and ϕ_{LC}.

6 Observations

$L =$ ______ mH, $C =$ ______ µF, $f_o = \dfrac{1}{2\pi\sqrt{LC}} =$ ______ kHz
Resistance of inductor = ______ Ω

Table 1: $R_1 =$ ______ Ω

Sl. No.	f (kHz)	V_i (V)	V_R (V)	V_R/V_i	V_R/V_i (Calculated)	ϕ_R	ϕ_R (Calculated)

Table 2: $R_2 =$ ______ Ω

Sl. No.	f (kHz)	V_i (V)	V_{LC} (V)	V_R/V_i	V_R/V_i (Calculated)

Table 3: $R_1 =$ ______ Ω

Sl. No.	f (kHz)	V_i (V)	V_{LC} (V)	V_{LC}/V_i	V_{LC}/V_i (Calculated)	ϕ_{LC}	ϕ_{LC} (Calculated)

7 Graphs

Plot the observed values of $V_R/V_i, V_{LC}/V_i, \phi_R$ and ϕ_{LC} versus frequency. Estimate the resonant frequency from graph in each case.

8 Results

9 Precautions

1. Make the ground connections carefully.
2. Make sure all the circuit connections are properly made Before supplying power supply